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SIMULATION OF OPTIMAL CONTROL OF ORBITAL VEHICLE THRUST DURING LAUNCH

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Abstract. In 21st century economically feasible and less expensive ways of space exploration and industrialization become the great challenge for mankind. One of the promising approaches is development of modern SSTO (single-stage-to-orbit) and VTOL (vertical-take-off-landing) technologies, leveraging latest achievements in new materials and propulsion systems. The paper presents results of simulation of dynamic systems, moving in central gravitational field, in an environment, where atmospheric drag force has a serious impact on system's dynamics. Model, proposed in paper, has been analyzed and build, based on Pontrjagin's principle of optimality. Optimal regular and singular thrust control of engine is analyzed. Research has been conducted on relations between optimal control feasibility and SSTO parameters, including initial acceleration and average (along trajectory) drag coefficient. Impact of drag dependence on Mach number on accuracy of computed optimal trajectory has been considered. One approach to solve the problem has been developed, where impact of proposed optimal control implementation on problem's optimality criteria is evaluated. The derived formula for calculating optimal control is invariant to conditions on right end of ascent trajectory.

INTRODUCTION

The classical Goddard problem – vertical launching to a maximum altitude [6] was the first case, when possibility of singular arcs – parts of active trajectory, when thrust is not maximal, has been considered. Physical reasons of optimal thrust not to be maximal, related to atmospheric drag force. During launch phase at some combination of velocity and altitude, drag force grows so high, that most of engine energy wasted on overcoming the drag force. Therefore it is important to find and build computational model to develop efficient launch trajectories, taking into consideration possibilty of singulac arcs. Particularly, we interested in problem of launching given payload on given orbit with maximum horizontal velocity and the end of launch phase. We consider flat trajectories in central newtonian gravitational field. Thrust vector is parallel to velocity vector, so we have one dimensional drag force. Control function is engine thrust.

Formal statement of the problem

Equations of motion are given as follows:

$$\dot{r} = v; \quad \dot{v} = (P(q)e\delta + F(r,v))/m + R(r); \quad \dot{m} = -q; \quad (1.1)$$

Here r – radius vector of mass center of the launching vehicle, v – vector of velocity, m – mass of the vehicle, $P(q)$ – engine thrust value, q – fuel mass consumption per time, e- unit vector of thrust direction, $F(r,v)$- drag force, and $R(r)$- gravitational force. For our convenience we introduce phase vector:

$$x = \{r,v,m\} \quad (1.2)$$

having dimensionality n

then the system 1.1 can be generally expressed as system of type:

$$\dot{x} = f_0(x) + \delta f_1(x) \quad (1.3)$$

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Here δ is control function, constrained by $\delta \in [0,1]$. Let's denote initial state of system 1.3 as x_0,

and final state as x_1. Problem now is stated as to find optimal control function *δ(t)* for system 1.3 moving from initial state x_0 to final state x_1 and providing maximum/minimum value to function $\varphi(x_1)$ (Mayer problem).

Problem Analysis

One of the well known approaches to this class of optimization problems is Pontrjagin's maximum principle. Hamiltonian for system 1.3 will be linear function of control *δ:*

$$H = H_0(x,\psi) + \delta H_1(x,\psi) \,, \qquad (1.4)$$

where ψ is conjugate vector. Giving that δ, according to maximum principle, should provide maximum to Hamiltonian 1.4 over optimal trajectory of system 1.3, one finds the following expressions:

$$\delta = \begin{cases} 1, & if\ H_1 > 0 \\ 0, & if\ H_1 < 0 \end{cases} \qquad (1.5)$$

In case when H_1=0 δ is undefined (singular case), part of trajectory, along which $H_1 = 0$, named singular arc. It is known, that along singular arc system 1.3 and conjugate system:

$$\dot{\psi} = -\frac{\partial H}{\partial x} \qquad (1.6)$$

have *2k+1* first integrals (*k* – named singularity level):

$$H_1 = 0, \dot{H}_1 = 0, \ddot{H}_1 = 0, \dots, H_1^{(2k-1)} = 0, H = 0 \qquad (1.7)$$

The last equality is valid only for problems with free final time. Left parts of integrals 1.7 are linear and homogeneous in regard to conjugate variables ψ. If *2k+1=n*, then system 1.7 represents equation for surface of singular control and method, proposed in [3] could be used for computation of optimal trajectories.

When *2k+1<n*, the method in [3] should be modified. In this case system 1.7 is to be resolved for *2k* conjugate variables and task of finding of remaining initial values of conjugate variables $\psi_i(0)$of boundary value problem could be reduced to problem of finding the beginning of singular arc *x(t)* and corresponding values of conjugate variables. This approach makes sense when there is sufficient information about control function at the beginning of launch trajectory.

Computational Algorithm

In particular case of our problem dimensionality *n = 4*. When *k =1*, one can express all conjugate variables as functions of one of them, for instance, ψ_1:

$$\psi_2 = \varphi_2(x)\psi_1; \quad \psi_3 = \varphi_3(x)\psi_1; \quad \psi_4 = \varphi_4(x)\psi_1; \qquad (1.8)$$

Index 1 should be assigned to one of conjugate variables, which is not equal 0 along the singular arc. Otherwise, relations 1.8 would make conjugate vector be 0, which contradicts to maximum principle. Equation for optimal control $\delta = \delta_0(t)$, having 1.8, becomes:

$$\ddot{H}_1 = \psi_1[A(x)\delta_0 - B(x)] = 0 \qquad (1.9)$$

So, δ_0 depends only on *x*, no conjugate variables involved:

$$\delta_0 = \frac{B(x)}{A(x)} \qquad (1.10)$$

It is reasonable to assume, that trajectory starts with $\delta = 1$ (maximum thrust), followed by, possibly, singular arc. This assumption allows one to build the following procedure for computation of optimal trajectory. Initially, equations of motion 1.3 are integrated with *δ=1* until complete fuel consumption and then computed trajectory has to be looked up to find interval *J*, where it is possible to start singular arc:

$$J = \{\, t_1 : \delta_0(t_1) \epsilon (0,1), A(x(t_1)) > 0 \,\} \qquad (1.11)$$

Inequality in 1.11 follows from Kelly necessary optimality condition in singular case [2]. Every point $t_1 \in J$ generated a corresponding set N_{t_1} of possible ends of singular arc:

$$N_{t_1} = \{\, t_2 : \delta_0(t_2) \epsilon (0,1),\ A\big(x(t_2)\big) > 0,\ m(t_2) \geq m_1 \,\} \qquad (1.12)$$

Here *m(t)* and m_1 are current and final mass of the launch vehicle, $x(t_2)$ computed by integrating system 1.3 with δ=1 while $t \in (0, t_1)$ and $\delta = \delta_0(t)$ for $t \epsilon [t_1, t_2]$. At moment t_2 control δ gets one of the boundary values: 0 or 1. When $t > t_2$ system 1.3 is integrated jointly with 1.6. Initial values for the latter are defined by formulae 1.8 when $= x(t_2)$, where constant $\psi_1(t_2)$ could be chosen equal to 1, because all equations involved in maximum principle are homogeneous. At this part of trajectory control is computed according to 1.5.

Above described computational schema completely defines trajectory by appropriate choice of parameters $t_1 \in J$, $t_2 \in N_{t_1}$and $\delta(t_2 + 0)$. The choice should satisfy to given boundary conditions and transversality conditions, that is, these parameters should minimize discrepancies between given boundary values and computed trajectory at end point.

Once the boundary conditions on the right end of trajectory have been satisfied, first part of launch trajectory, preceding to singular arc, has to be computed. Equations of motion 1.3 and conjugate system are integrated backward from point $x(t_1)$, $\psi(t_1)$ to start point.

Numerical Computations

Parameters for numerical estimates have been chosen, based on specifications of Space Launch System, being developed by NASA. Equations of motion 1.1, written in flat polar coordinate system, become like these:

$$\dot{r} = v\sin(\theta);$$

$$\dot{v} = \frac{a_0[1-p_h(r)]\delta - F(r,v)}{m} - \frac{\sin(\theta)}{r^2}; \qquad (1.13)$$

$$\dot{\theta} = \left(v - \frac{1}{rv}\right)\frac{\cos(\theta)}{r};$$

$$\dot{m} = -a_0 c\delta;$$

Here r is a distance between center of mass of vehicle and center of gravity (Earth), v – vehicle velocity, θ – angle between vector of velocity and local horizon, δ – control, a_0- initial thrust to weight ratio, $p_h(r)$ – atmospheric counter pressure on engine nuzzle, $F(r,v)$ – atmospheric drag force. Initial conditions have been picked up, assuming that trajectory starts with vertical takeoff with maximum thrust, so the computation starts at moment of time, when $\theta_0 < \frac{\pi}{2}$ and $v_0 > 0$

Series of computational experiments with slight variations of initial values v_0, θ_0 have shown, indeed, that singular arcs are possible. Figure 1 shows diagram of control function behavior. What is interesting, that singular arc visibly occupies most part of active trajectory (where ≥ 0).

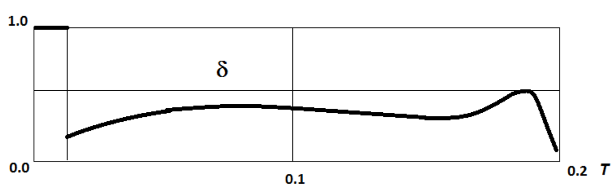

Figure 1.

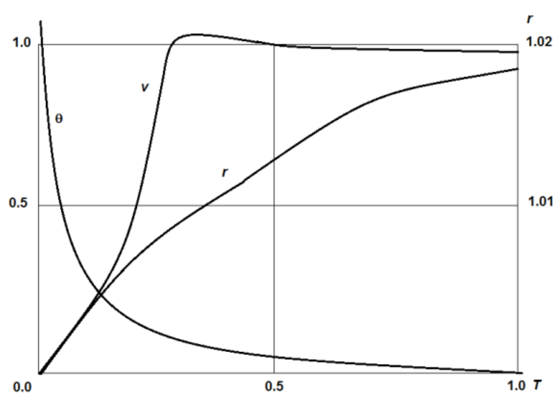

Figure 2. Depicts time diagrams of main coordinates of trajectory.

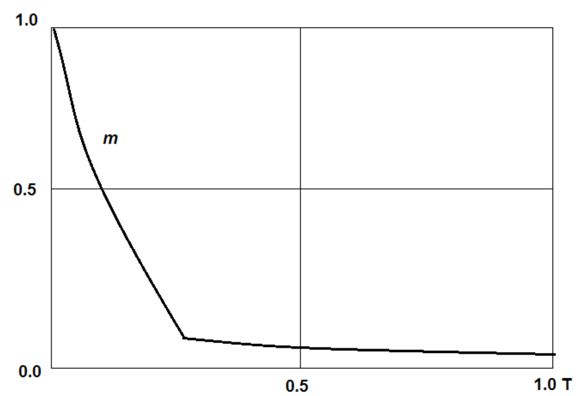

Figure 3. Diagram of vehicle mass over time.

Summary

The purpose of this work was to present a feasible technique for computation of optimal trajectories, including singular arcs. Computational experiments have proved that such singular arcs can exist. Further development aims at finding, how effective might be singular arcs, depending on vehicle structural parameters or engine capabilities.

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